Optimal Ordering and Trade Credit Policy for EOQ Model

Hardik Soni* and Nita H. Shah**

ABSTRACT

Trade credit is the most prevailing economic phenomena used by the suppliers for encouraging the retailers to increase their ordering quantity. In this article, an attempt is made to derive a mathematical model to find optimal credit policy and hence ordering quantity to minimize the cost. Even though, credit period is offered by the supplier, both parties (supplier and retailer) sit together to agree upon the permissible credit for settlement of the accounts by the retailer. A numerical example is given to support the analytical arguments.

JEL. Classification: C02; C61

Key words: Trade Credit, Optimal ordering quantity, Lot-size

1. INTRODUCTION

The classical EOQ model is based on the assumption that the retailer must pay for the items as soon as it is received by the system. However, the most prevailing practice is that the supplier may offer a credit period to the retailer to settle his account within the allowable settlement period. The supplier will vary terms in anticipation of capturing new business, to attract specific group of customers to achieve marketing goals i.e. for supplier who offers trade credit, it is an effective means of price discrimination as well as efficient tool to stimulate the demand of his products.

Haley and Higgins (1973) studied the interaction between inventory policy and trade credit in the context of the classical lot – size model. Goyal (1985) developed mathematical model when supplier offers permissible credit period to settle the account, so that no interest charges are payable from the outstanding amount if the

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The above stated articles assumed that trade credit is constant even though most of the articles stated that allowable trade credit can be considered as demand increasing phenomena. In this article, an attempt is made to derive optimal trade credit and ordering policy for the retailer to minimize the total cost of the inventory system. It is established that the total cost per time unit of an inventory system is a function of credit period. The analytical results are supported by a numerical example.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions are used to aforesaid model:

1. The inventory system deals with the single item.
2. The demand is \( R = \alpha M^\beta \); \( \alpha, \beta > 0, \alpha \neq \beta \), \( M \) denotes trade credit offered by the supplier and a decision variable.
3. Shortages are not allowed and lead time is zero.
4. Replenishment is instantaneous.
5. Replenishment rate is infinite.
6. If the retailer pays by \( M \), then supplier does not charge any interest. If the retailer pays after \( M \), he can keep the difference in the unit sale price and unit cost in an interest bearing account at the rate of \( Ie/\text{unit/year} \).

The notations are as under:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>The inventory holding cost/unit/year excluding interest charges.</td>
</tr>
<tr>
<td>( p )</td>
<td>The selling price/unit.</td>
</tr>
<tr>
<td>( C )</td>
<td>The unit purchase cost, with ( C &lt; p ).</td>
</tr>
<tr>
<td>( A )</td>
<td>The ordering cost/order.</td>
</tr>
<tr>
<td>( M )</td>
<td>The credit period in settling the account. (a decision variable)</td>
</tr>
<tr>
<td>( T )</td>
<td>The replenish cycle time (a decision variable)</td>
</tr>
<tr>
<td>( Ic )</td>
<td>The interest charged per $ in stock per year by the supplier when retailer pays during ([M, T] ).</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>Ie</td>
<td>The interest earned/$/year.</td>
</tr>
<tr>
<td>IHC</td>
<td>Inventory holding cost/time unit.</td>
</tr>
<tr>
<td>PC</td>
<td>Purchase cost / time unit.</td>
</tr>
<tr>
<td>OC</td>
<td>Ordering cost / time unit.</td>
</tr>
<tr>
<td>IE</td>
<td>Interest earned / time unit.</td>
</tr>
<tr>
<td>IC</td>
<td>Interest charged / time unit.</td>
</tr>
<tr>
<td>(t)</td>
<td>The on-hand inventory level at time t (0 ≤ t ≤ T).</td>
</tr>
<tr>
<td>K_i(T)</td>
<td>The total cost of an inventory system per time unit, i = 1, 2.</td>
</tr>
</tbody>
</table>

### 3. MATHEMATICAL FORMULATION

The on-hand inventory depletes due to demand R(M). The instantaneous state of inventory at any instant of time t, 0 ≤ t ≤ T is governed by the differential equation

\[
\frac{dQ(t)}{dt} = -R(M) = -\alpha M^\beta, \quad 0 \leq t \leq T
\]

(3.1)

with initial condition \( Q(0) = Q \) and boundary condition \( Q(T) = 0 \). Consequently, the solution of (3.1) is given by

\[
Q(t) = R(M)(T - t); \quad 0 \leq t \leq T
\]

(3.2)

and the order quantity is \( Q = R(M)T \)

(3.3)

The cost components per unit time are as follows:

- Ordering cost; \( OC = \frac{A}{T} \)

(3.4)

- Inventory holding cost;

\[
IHC = \frac{h}{T} \int_0^T Q(t) dt = \frac{hR(M)T}{2}
\]

(3.5)

Regarding interest charged and earned, based on the length of the cycle time T, two cases arise:

Case 1: \( T \leq M \)

Case 2: \( M < T \)

We discuss each case in detail.

**Case 1: \( T \leq M \)**

Inventory level

![Fig. 1 T ≤ M](image)

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Here, the retailer sells Q-units during [0, T] and is paying CR(M)T in full to the supplier at time $M \geq T$. So interest charges are zero. i.e. $IC_1 = 0$.

(3.6)

The retailer sells products during [0, T] and deposits the revenue in an interest bearing account at the rate of $Ie$/year. In the period, [T, M] the retailer deposits revenue into the account that earns $Ie$/year. Therefore total interest earned per time unit is

$$IE_1 = \frac{pIe}{T} \left[ \int_0^T R(M) t \, dt + R(M)T(M - T) \right] = \frac{pIe R(M)(2M - T)}{2}$$

(3.7)

Hence, the total cost; $K_1(T, M)$ per time unit of an inventory system is given by

$$K_1(T, M) = OC + IHC + IC_1 - IE_1$$

(3.8)

Here, T and M are continuous decision variables. The optimal values of M and T can be obtained by solving

$$\frac{\partial K_1(T, M)}{\partial M} = \frac{hT\alpha M^\beta}{2} - \frac{pIe}{2} \alpha M^{\beta-1} \beta(2M - T) - pIe \alpha M^\beta = 0 \quad (3.9a)$$

$$\frac{\partial K_1(T, M)}{\partial T} = -\frac{A}{T^2} + \frac{(h+pIe)\alpha M^\beta}{2} = 0 \quad (3.9b)$$

simultaneously by suitable numerical method.

The obtained M and T minimizes the total cost $K_1(T, M)$; provided $XY - Z^2 > 0$,

$$X = \frac{\partial^2 K_1(T, M)}{\partial^2 T} = \frac{2A}{T^2},$$

$$Z = \frac{\partial^2 K_1(T, M)}{\partial M \partial T} = \frac{(h+pIe)\alpha M^{\beta-1} \beta}{2}$$

(3.9c)
\[
Y = \frac{\partial^2 K_1(T, M)}{\partial M^2} = \frac{hT\alpha M^{\beta-2} \beta(\beta-1)}{2} - \frac{p \cdot I_0 \cdot \alpha M^{\beta-2} \beta(\beta-1)(2M - T)}{2} - 2p \cdot I_0 \cdot \alpha M^{\beta-1} \beta
\]

**Case 2: \( M < T < N \)**

Inventory level

The retailer sells units at selling price \( p \) $/unit and deposits the revenue into an interest bearing account at an interest rate \( I_0 \)/unit/annum during \([0, M]\). Therefore, interest earned during \([0, M]\) is given by

\[
IE_2 = \frac{p \cdot I_0 \cdot M}{T} \int_0^M R(M) t \, dt = \frac{p \cdot I_0 \cdot \alpha \cdot M^{\beta+2}}{2T}
\]

and during \([M, T]\) supplier will charge interest rate at \( I_c \)/unit/annum. So total interest charged per time unit during \([M, T]\) is

\[
IC_2 = \frac{Clc}{T} \int_M^T Q(t) \, dt = \frac{ClcR(M)(T - M)^2}{2T}
\]

The total cost, \( K_2(T, M) \) per time unit of an inventory system is given by

\[
K_2(T, M) = OC + IHC + IC_2 - IE_2
\]

The optimal values of \( M \) and \( T \) can be obtained by simultaneously solving
Optimal Ordering and Trade Credit Policy for EOQ Model

\[ \frac{\partial K_2(T, M)}{\partial M} = \frac{hT}{2} \alpha M^{\beta-1} + \frac{C Ic}{2T} \alpha M^{\beta-1} \beta (T - M)^2 - \frac{C Ic}{T} \alpha M^{\beta} (T - M) \]

\[ - \frac{p Ie}{2T} \alpha M^{\beta+1} (\beta + 2) = 0 \]  
(3.12a)

\[ \frac{\partial K_2(T, M)}{\partial T} = - \frac{A}{T^2} + \frac{h}{2} \alpha M^{\beta} + \frac{C Ic}{2T^2} \alpha M^{\beta} (T^2 - M^2) + \frac{p Ie}{2T^2} \alpha M^{\beta+2} = 0 \]  
(3.12b)

The obtained \( M \) and \( T \) minimizes the total cost \( K_2(T, M) \); provided

\[ EF - G^2 > 0 \]  
(3.12c)

where,

\[ E = \frac{\partial^2 K_2(T, M)}{\partial T^2} = \frac{2A}{T^3} + \frac{C Ic}{T} \alpha M^{\beta} - \frac{C Ic}{T^3} \alpha M^{\beta} (T^2 - M^2) - \frac{p Ie}{T^3} \alpha M^{\beta+2} \]

\[ F = \frac{\partial^2 K_2(T, M)}{\partial M^2} = \frac{hT}{2} \alpha M^{\beta-2} \beta (\beta - 1) + \frac{C Ic}{2T} \alpha M^{\beta-2} \beta (\beta - 1) (T - M)^2 \]

\[ - \frac{2C Ic}{T} \alpha M^{\beta-1} \beta (T - M) + \frac{C Ic}{2T} \alpha M^{\beta} - \frac{p Ie}{2T} \alpha M^{\beta} (\beta + 1) (\beta + 2) \]

\[ G = \frac{\partial^2 K_2(T, M)}{\partial M \partial T} = \frac{h}{2} \alpha M^{\beta-1} \beta + \frac{C Ic}{2T^2} \alpha M^{\beta-1} \beta (T^2 - M^2) - \frac{C Ic}{T^2} \alpha M^{\beta+1} \]

\[ + \frac{p Ie}{2T^2} \alpha M^{\beta+1} (\beta + 2) \]

4. SOME RESULTS

**Proposition 4.1:** \( K_i(T, M) \) is decreasing function of \( M \) \((i = 1, 2)\).

**Proof:** Clearly,

\[ \frac{\partial K_1(T, M)}{\partial M} = \frac{hT \alpha M^{\beta}}{2} - \frac{p Ie \alpha M^{\beta-1} \beta (2M - T)}{2} - p Ie \alpha M^{\beta} < 0 \]
\[
\frac{\partial K_2(T, M)}{\partial M} = h \frac{T}{2} M^{\beta - 1} \beta + \frac{C Ic}{2T} \alpha M^{\beta - 1} \beta (T - M)^2 - \frac{C Ic}{T} \alpha M^\beta (T - M) \\
- \frac{p Ie}{2T} M^{\beta + 1} (\beta + 2) < 0
\]

**Proposition 4.2:** \( K_i(T, M) \) is deceasing function of \( T \) (i = 1, 2).

**Proof:** Clearly,

\[
\frac{\partial K_1(T, M)}{\partial T} = - \frac{A}{T^2} + \frac{(h + p Ie)\alpha M^\beta}{2} < 0
\]

\[
\frac{\partial K_2(T, M)}{\partial T} = - \frac{A}{T^2} + \frac{h \alpha M^\beta}{2} + \frac{C Ic}{2T} \alpha M^\beta (T^2 - M^2) \\
+ \frac{p Ie}{2T} M^{\beta + 2} < 0
\]

**Proposition 4.3:** \( K_i(T, M) \) is increasing function of \( \alpha \) (i = 1, 2).

**Proof:** Clearly,

\[
\frac{\partial K_1(T, M)}{\partial \alpha} = \frac{h T M^\beta}{2} - \frac{p Ie}{2} M^\beta (2M - T) > 0
\]

\[
\frac{\partial K_2(T, M)}{\partial \alpha} = \frac{h T M^\beta}{2} + \frac{C Ic}{2T} M^\beta (T - M)^2 - \frac{p Ie M^{\beta + 2}}{2T} > 0
\]

**Proposition 4.4:** \( K_i(T, M) \) is decreasing function of \( \beta \) (i = 1, 2).

**Proof:** Clearly,

\[
\frac{\partial K_1(T, M)}{\partial \beta} = \frac{\alpha M^\beta \ln(M)}{2} \left( h T - p Ie (2M - T) \right) < 0
\]

\[
\frac{\partial K_2(T, M)}{\partial \beta} = \frac{\alpha M^\beta \ln(M)}{2T} \left( h T^2 - C Ic (T - M)^2 - p Ie M^2 \right) < 0
\]

In the next section, computation flow chart is given to search for optimal solution.
5. FLOW CHART

Start

Compute T and M from Case-1

Is T < M

Yes → Calculate K₁(T, M)

No → Compute T and M from Case-2

Calculate K₂(T, M)

Stop
6. NUMERICAL EXAMPLE

Consider following parametric values:

\[ [A, C, h, Ie, p, Ic] = [200, 20, 2, 0.12, 30, 0.18] \]

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \alpha )</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1000</td>
<td>T = 0.3229</td>
<td>T = 0.2662</td>
<td>T = 0.2321</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M = 0.0228</td>
<td>M = 0.0188</td>
<td>M = 0.0164</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R = 685.25</td>
<td>R = 1008.22</td>
<td>R = 1326.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q = 221.24</td>
<td>Q = 268.36</td>
<td>Q = 307.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( K_2(T,M) = 1172.62 )</td>
<td>( K_2(T,M) = 1434.49 )</td>
<td>( K_2(T,M) = 1645.10 )</td>
</tr>
<tr>
<td>0.2</td>
<td>1000</td>
<td>T = 0.3628</td>
<td>T = 0.3018</td>
<td>T = 0.2648</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M = 0.0470</td>
<td>M = 0.0391</td>
<td>M = 0.0343</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R = 542.60</td>
<td>R = 784.45</td>
<td>R = 1018.93</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q = 196.87</td>
<td>Q = 236.71</td>
<td>Q = 269.78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( K_2(T,M) = 1010.59 )</td>
<td>( K_2(T,M) = 1215.11 )</td>
<td>( K_2(T,M) = 1384.86 )</td>
</tr>
<tr>
<td>0.3</td>
<td>1000</td>
<td>T = 0.3972</td>
<td>T = 0.3330</td>
<td>T = 0.2938</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M = 0.0713</td>
<td>M = 0.0598</td>
<td>M = 0.0527</td>
</tr>
<tr>
<td></td>
<td></td>
<td>R = 452.80</td>
<td>R = 644.21</td>
<td>R = 827.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q = 179.84</td>
<td>Q = 214.51</td>
<td>Q = 243.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( K_2(T,M) = 890.90 )</td>
<td>( K_2(T,M) = 1062.66 )</td>
<td>( K_2(T,M) = 1204.24 )</td>
</tr>
</tbody>
</table>

7. CONCLUSION

In this paper, an attempt is made to develop an EOQ model in which demand is assumed to be increasing function of credit period (a decision variable) when supplier offers a credit period, if retailer could not settle his account. Increase in fixed partial demand decreases trade credit and increases annual demand significantly. Exponent increase in demand increases trade credit and decreases annual demand significantly. An easy – to – use computational flow-chart is given to search for optimal policy. The observed managerial issues are as follows:

(1) Increase in fixed partial demand increases the order quantity and total cost of an inventory system.
(2) Increase in exponent factor \( \beta \) decreases the order quantity and total cost of an inventory system.

REFERENCES


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